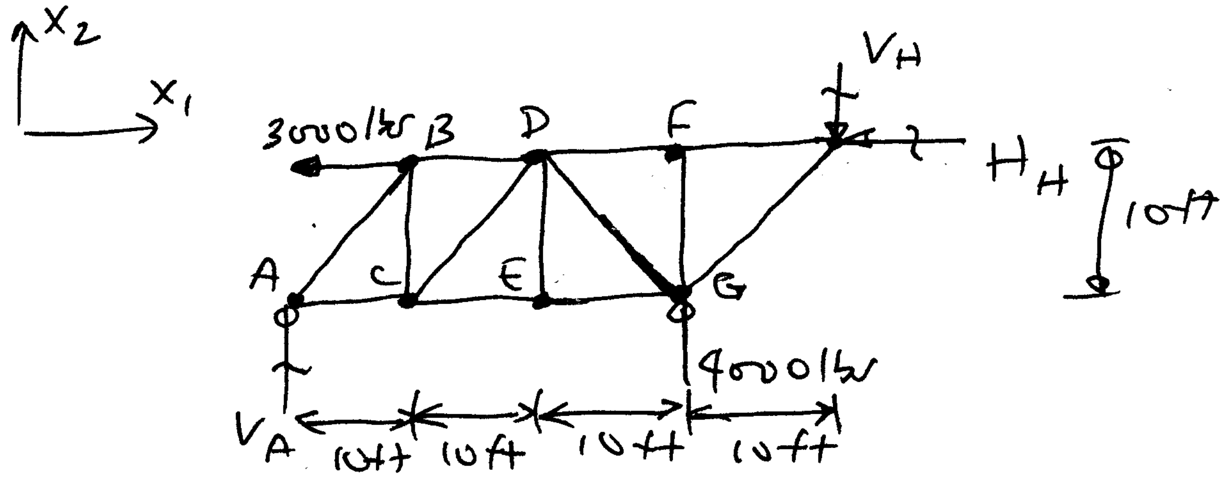
PAL
9/29/07

Unified Engineering Problem Set
Week 5   Fall, 2007

SOLUTIONS

M5.1

(a) Draw the Free Body Diagram



(b) Determine the reaction forces by applying the equilibrium equations:

$\sum F_1 = 0 \xrightarrow{+} \Rightarrow -H_H - 3000\text{ lbs} = 0$

$\Rightarrow \boxed{H_H = -3000\text{ lbs}}$

$\sum F_2 = 0 \uparrow + \Rightarrow V_A - V_H + 4000\text{ lbs} = 0$

$\sum M = 0 \circlearrowright + \Rightarrow -V_A(40\text{ ft}) - 4000\text{ lbs}(10\text{ ft}) = 0$
(about point H)

$\Rightarrow \boxed{V_A = -1000\text{ lbs}}$

using the $\sum F_2 = 0$ equation:

$$-1000\text{ lbs} - V_H + 4000\text{ lbs} = 0$$

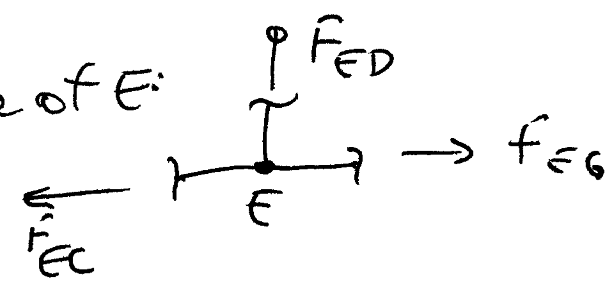
$$\Rightarrow \boxed{V_H = 3000\text{ lbs}}$$

Summarizing:

$$\boxed{\begin{aligned} V_A &= -1000\text{ lbs} \\ V_H &= 3000\text{ lbs} \\ H_H &= -3000\text{ lbs} \end{aligned}}$$

(c) <u>YES!</u>. Consider point E and point F and vertical (x_2) equilibrium at each point.

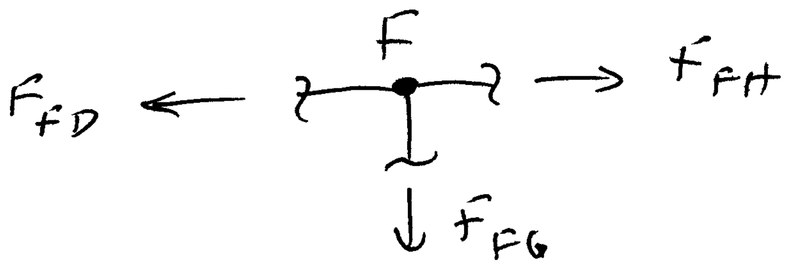
For the case of E:



Since only the bar ED has a vertical direction, only it can carry a vertical load (x_2-direction). There is <u>no load</u> applied at point E, so this gives $\boxed{F_{ED} = 0}$ by equilibrium in the vertical (x_2) direction.

The same argument can be made at point F:

Drawing gives:

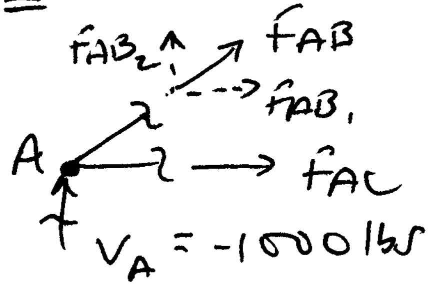


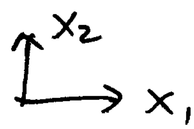
Again vertical (x_2) equilibrium gives

$$\boxed{f_{FG} = 0}$$

(d) Using the Method of Joints, choose an end from which to start.

Choose A here:





Note that the diagonal bars make 45° angles to the x_1 and x_2 axes since the truss unit is a square (10 ft × 10 ft).

Thus, the x_1 and x_2 force components of the diagonal bars are:

x_1 : $f_{Bar} \cos 45° = f_{Bar}\,(0.707)$

x_2 : $f_{Bar} \sin 45° = f_{Bar}\,(0.707)$

Now use equilibrium:

$$\sum F_1 = 0 \xrightarrow{+} \Rightarrow 0.707 F_{AB} + F_{AC} = 0$$

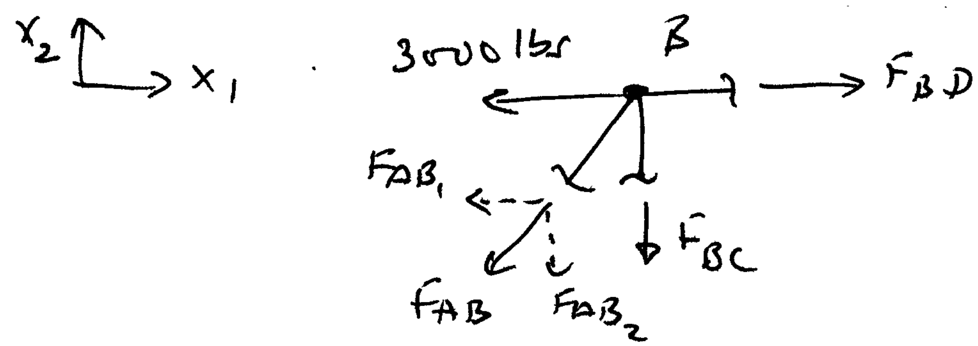
$$\sum F_2 = 0 \; \uparrow + \Rightarrow -1000 \text{ lbs} + 0.707 F_{AB} = 0$$

$$\Rightarrow \boxed{F_{AB} = 1414 \text{ lbs}}$$

first equation gives:

$$\boxed{F_{AC} = -1000 \text{ lbs}}$$

Proceed to Joint B



$$\sum F_1 = 0 \xrightarrow{+} \Rightarrow -3000 \text{ lbs} + F_{BD} - 0.707 F_{AB} = 0$$

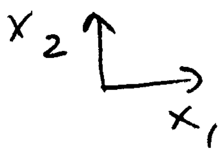
using $F_{AB} = 1414$ lbs gives:

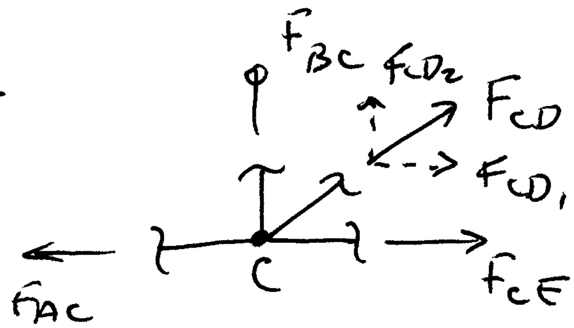
$$\boxed{F_{BD} = 4000 \text{ lbs}}$$

$$\sum F_2 = 0 \; \uparrow + \Rightarrow -0.707 F_{AB} - F_{BC} = 0$$

$$\Rightarrow \boxed{F_{BC} = -1000 \text{ lbs}}$$

to <u>Joint C</u>





$$\Sigma F_1 = 0 \xrightarrow{+} \Rightarrow -F_{AC} + F_{CE} + 0.707 F_{CD} = 0$$

using $F_{AC} = -1000$ lbs gives:

$$1000\,\text{lbs} + F_{CE} + 0.707 F_{CD} = 0$$

Also use $\Sigma F_2 = 0 \uparrow + \Rightarrow F_{BC} + 0.707 F_{CD} = 0$

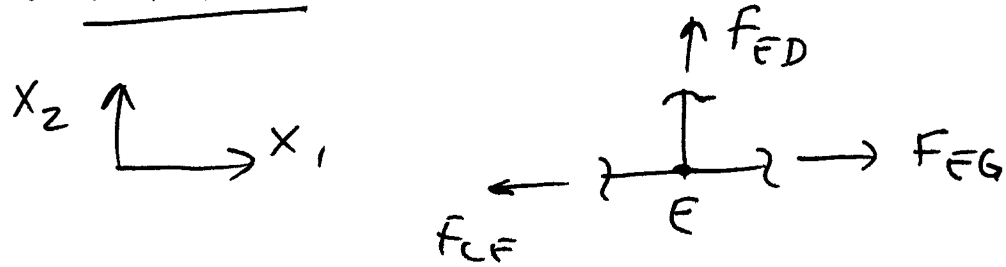
using $F_{BC} = -1000$ lbs gives:

$$\boxed{F_{CD} = 1414\ \text{lbs}}$$

use this in the $\Sigma F_1 = 0$ equation to get

$$\boxed{F_{CE} = -2000\ \text{lbs}}$$

to <u>Joint E</u>



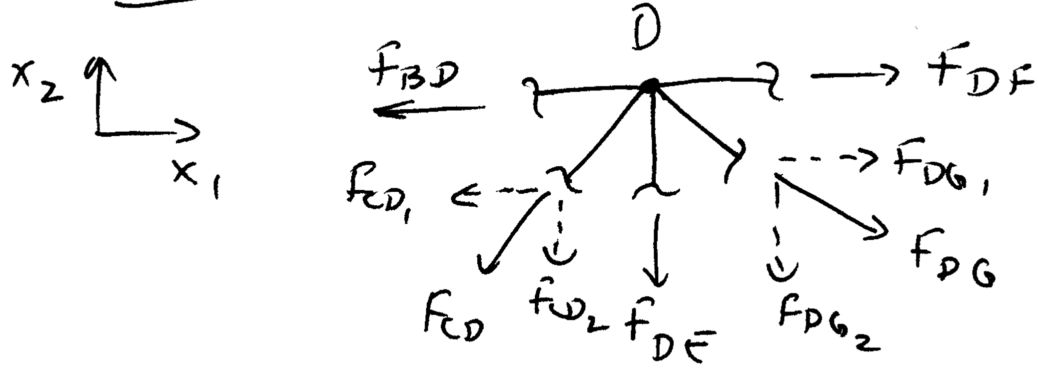
$$\Sigma F_1 = 0 \xrightarrow{+} \Rightarrow -F_{CE} + F_{EG} = 0$$

using $F_{CE} = -2000$ lbs gives:

$$\boxed{F_{EG} = -2000\ \text{lbs.}}$$

$$\Sigma F_2 = 0 \uparrow + \Rightarrow \boxed{F_{ED} = 0}$$ (as in part (c))

to Joint D



$$\sum F_1 = 0 \xrightarrow{+} \Rightarrow -F_{BD} + F_{DF} - 0.707\, F_{CD} + 0.707 F_{DG} = 0$$

have $F_{BD} = 4000$ lbs, $F_{CD} = 1414$ lbs

gives: $-5000 \text{ lbs} + F_{DF} + 0.707 F_{DG} = 0$

with...

$$\sum F_2 = 0 \uparrow + \Rightarrow -0.707 F_{CD} - F_{DE} - 0.707 F_{DG} = 0$$

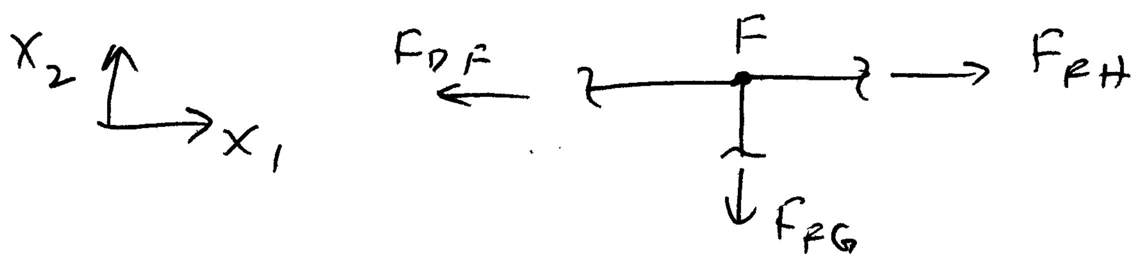
already have $F_{DE} = 0$ and $F_{CD} = 1414$ lbs,

so: $\boxed{F_{DG} = -1414 \text{ lbs}}$

and using in the $\sum F_1 = 0$ equation:

$$\Rightarrow \boxed{F_{DF} = 6000 \text{ lbs}}$$

to Joint F

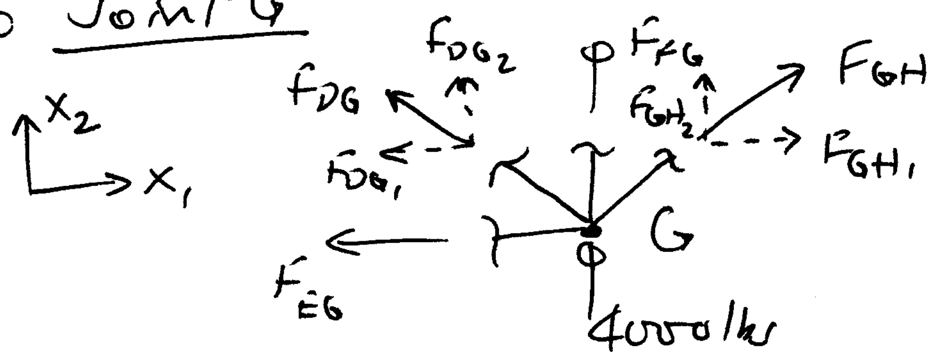


$\sum F_1 = 0 \xrightarrow{+} \Rightarrow -F_{DF} + F_{FH} = 0$

with $F_{DF} = 6000$ lbs $\Rightarrow$ $\boxed{F_{FH} = 6000 \text{ lbs}}$

$\sum F_2 = 0 \uparrow + \Rightarrow$ $\boxed{F_{FG} = 0}$ (as in part (c))

to <u>Joint G</u>



$\sum F_1 = 0 \xrightarrow{+} \Rightarrow -F_{EG} - 0.707\, F_{DG} + 0.707\, F_{GH} = 0$

already have $F_{EG} = -2000$ lbs, $F_{DG} = -1414$ lbs

giving $\boxed{F_{GH} = -4242 \text{ lbs}}$

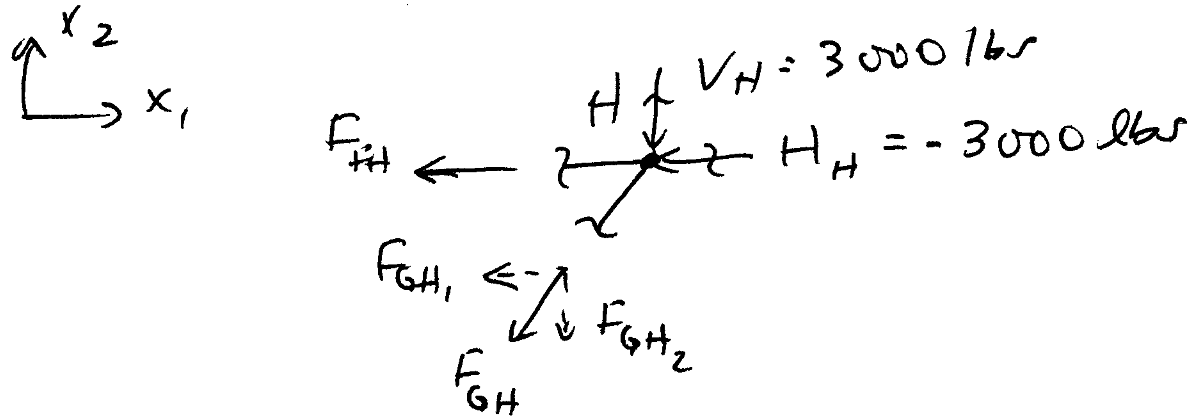
$\sum F_2 = 0 \uparrow + \Rightarrow 4000 \text{ lbs} + 0.707\, F_{DG} + F_{FG} + 0.707\, F_{GH} = 0$

add on knowledge of $F_{FG} = 0$ and can check on F_{GH}:

$4000 \text{ lbs} - 1000 \text{ lbs} - 3000 \text{ lbs} \stackrel{?}{=} 0$

YES

finally to <u>Joint H</u>:



This allows us to do final checks....

$\sum F_1 = 0 \xrightarrow{+} \Rightarrow -F_{FH} - (-3000 \text{ lbs}) - 0.707 F_{GH} = 0$

using $F_{FH} = 6000$ lbs, $F_{GH} = -4242$ lbs

gives:

$-6000 \text{ lbs} + 3000 \text{ lbs} + 3000 \text{ lbs} \overset{?}{=} 0$

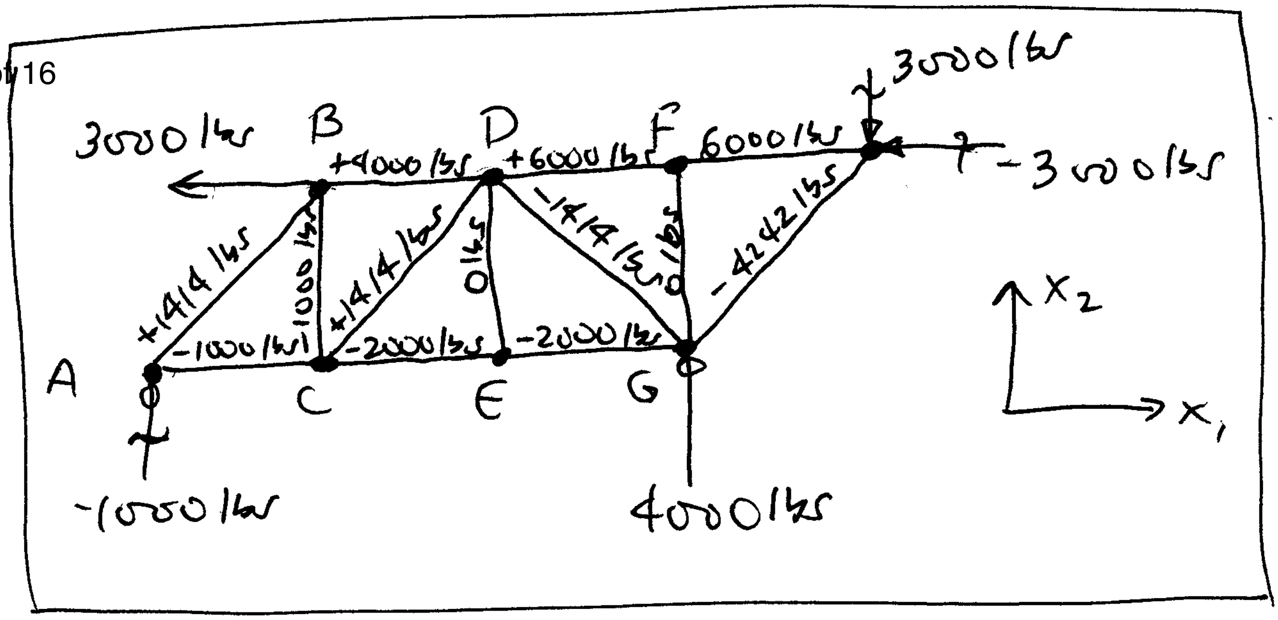
<u>YES</u> checks ✓

$\sum F_2 = 0 \; \uparrow + \Rightarrow -3000 \text{ lbs} - 0.707 F_{GH} \overset{?}{=} 0$

gives: $-3000 \text{ lbs} + 3000 \text{ lbs} \overset{?}{=} 0$

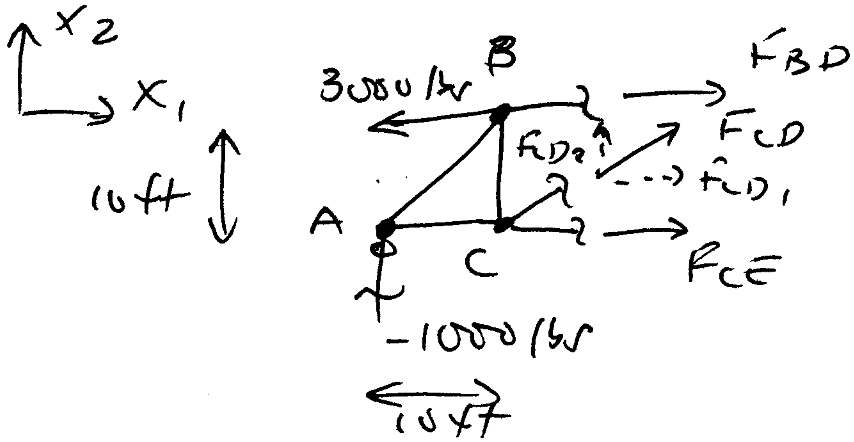
<u>YES</u> checks ✓

Summarize by drawing the truss and placing the bar load above each bar with (+) tension and (−) compression.



(e) To determine the loads in the bars of the second bay (BD, CD, CE) by the Method of Sections, a "sectional cut" must be made that goes through those bars. There must be only three bars of unknown load so that the three equilibrium equations yield a determinate solution.

Choose the following cut from the end of point A (the same could be done for point H, but less external loads/reactions are involved here):



Apply the equations of equilibrium:

$\Sigma F_1 = 0 \ \xrightarrow{+} \ \Rightarrow -3000\,lbs + F_{BD} + F_{CE} + 0.707 F_{CD} = 0$

$\Sigma F_2 = 0 \ \uparrow + \ \Rightarrow -1000\,lbs + 0.707 F_{CD} = 0$

$\Rightarrow \boxed{F_{CD} = 1414\,lbs}$ ✓ checks

$\Sigma M = 0$ (about point C) $\circlearrowleft + \Rightarrow (3000\,lbs)(10\,ft) - (-1000\,lbs)(10\,ft)$

$- F_{BD}(10\,ft) = 0$

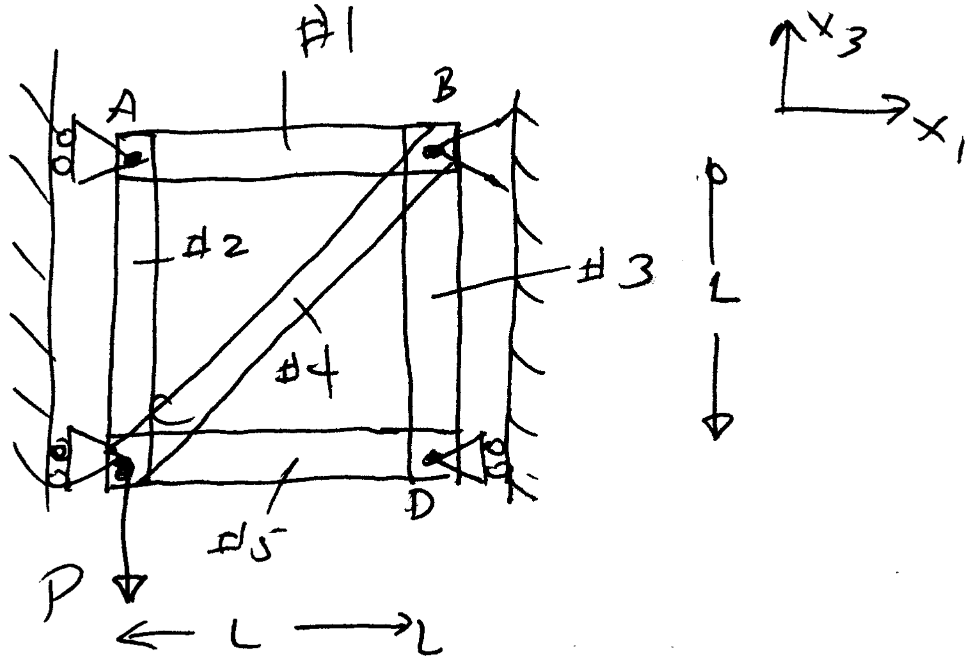
$\Rightarrow \boxed{F_{BD} = 4000\,lbs}$ ✓ checks

Putting results in $\Sigma F_1 = 0$ equation:

$-3000\,lbs + 4000\,lbs + F_{CE} + 1000\,lbs = 0$

$\Rightarrow \boxed{F_{CE} = -2000\,lbs}$ ✓ checks

M5.2



(a) Manifestation of "Compatibility of Displacement" in this case is $\boxed{\text{Compatibility at the joints}}$. All items connected at a joint must displace the same way at the joint

First define the displacement at each joint:

B is pinned, so $\delta_B = 0$

A, C, and D are rollers, so they only have displacement in x_3: δ_A, δ_C, δ_D

Define the change in length (overall deflection of each bar as δ_{bar}. This can be equated to the displacements at the joints:

Bar #2 moves only along x_3. So:

$$\delta_2 = \delta_C - \delta_A \qquad (1)$$

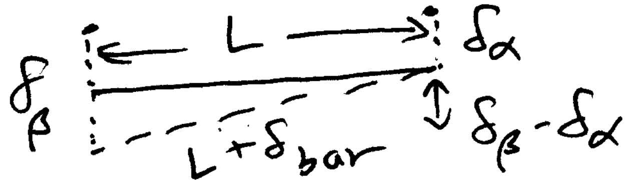
Bar #3 moves only along x_3. So:

$$\delta_3 = \delta_D - \delta_B$$

but with $\delta_B = 0$

$$\delta_3 = \delta_D \qquad (2)$$

In looking at the other three bars, they have endpoints that move only in x_3 but are at different x_1 locations. So one needs to look at the right triangle of a generic bar with endpoints that can move



Looking at this right triangle, squaring sides gives:

$$L^2 + (\delta_\beta - \delta_\alpha)^2 = (L + \delta_{bar})^2$$

This works for Bar #1 and Bar #5 - - - -

<u>Bar #1</u>: $\delta_\alpha = \delta_B = 0$; $\delta_\beta = \delta_A$

so:
$$L^2 + \delta_A^2 = L^2 + 2L\delta_1 + \delta_1^2$$
$$\Rightarrow \delta_A^2 = 2L\delta_1 + \delta_1^2 \quad (3)$$

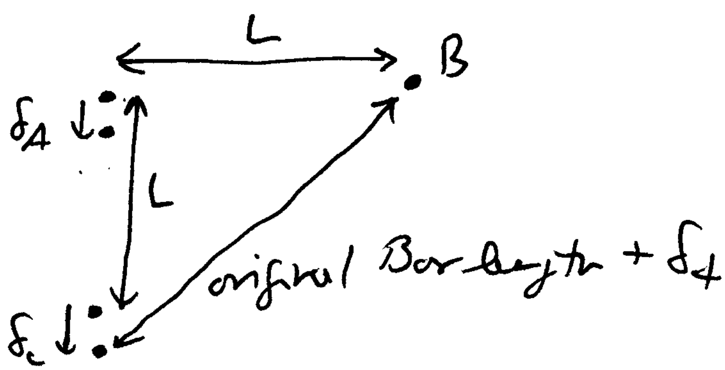
<u>Bar #5</u>: $\delta_\alpha = \delta_D$; $\delta_\beta = \delta_c$

so:
$$L^2 + \delta_D^2 - 2\delta_c\delta_D + \delta_c^2 = L^2 + 2L\delta_5 + \delta_5^2$$
$$\Rightarrow \delta_D^2 - 2\delta_c\delta_D + \delta_c^2 = 2L\delta_5 + \delta_5^2 \quad (4)$$

Finally look at the special case of Bar #4 and expand on this triangle.

<u>Bar #4</u>

one leg is the distance between the walls L



The other leg is originally from A to C, a distance of the basic length L. This changes by the deformation of joint C to become: $(L + \delta_c)$

Finally, the original length of Bar #4 is $\sqrt{L^2 + L^2} = 1.414L$

so its new length is $(1.414L + \delta_4)$

Putting all this together as the parts of the right triangle gives:

$$L^2 + (L + \delta_c)^2 = (1.414L + \delta_4)^2$$

working this:

$$L^2 + L^2 + 2\delta_c L + \delta_c^2 = 2L^2 + 2.828L\delta_4 + \delta_4^2$$

$$\Rightarrow \quad 2L\delta_c + \delta_c^2 = 2.828L\delta_4 + \delta_4^2 \qquad (5)$$

Summarizing, the working equations of the Compatibility of Displacement are:

$$\boxed{\begin{array}{ll} \delta_2 = \delta_c - \delta_A & (1) \\ \delta_3 = \delta_D & (2) \\ \delta_A^2 = 2L\delta_1 + \delta_1^2 & (3) \\ \delta_D^2 - 2\delta_c\delta_D + \delta_c^2 = 2L\delta_5 + \delta_5^2 & (4) \\ 2L\delta_c + \delta_c^2 = 2.828L\delta_4 + \delta_4^2 & (5) \end{array}}$$

(5) If it is assumed that the deflections are small, any higher order terms (H.O.T.) in the deflections can be linearized by considering first order effects.

→ There are no changes for Bar #2 or Bar #3

For Bar #1, one can equate the joint deflection with that of the bar:

$$\delta_1 = \delta_A \qquad (3)'$$

For Bar #5, a similar linearization is made with the bar length change being equal to the difference in joint deflections:

$$\delta_5 = \delta_D - \delta_C \qquad (4)'$$

For Bar #4, higher order terms in equation (5) can be ignored to get:

$$2L\delta_c = 2.828 L \delta_4$$

$$\Rightarrow \delta_c = 1.414\, \delta_4 \qquad (5)'$$

Summarizing the linearized equations:

$$\boxed{\begin{aligned}
\delta_2 &= \delta_c - \delta_A && (1)\\
\delta_3 &= \delta_D && (2)\\
\delta_1 &= \delta_A && (3)'\\
\delta_5 &= \delta_D - \delta_c && (4)'\\
\delta_c &= 1.414\,\delta_4 && (5)'
\end{aligned}}$$

(NOTE: There are different ways to linearize the equations and this will result in different overall results. Only full inclusion results in the actual results....)

(c) "Small" deflections allow a linearization of equations to take place. This then allows a linear solution of the full set of equations. If the deflections are "large" and the higher order (nonlinear) equations must be used, the overall set of governing equations are quite nonlinear and would need to be solved using numerical approximations or other techniques